B.M.S COLLEGE FOR WOMEN AUTONOMOUS
BENGALURU – 560004

END SEMESTER EXAMINATION – OCTOBER 2022

M.Sc. in Mathematics – II Semester

Complex Analysis

Course Code: MM202T | **QP Code: 21002**
Duration: 3 Hours | **Max marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. (a) Define harmonic functions and evaluate $\int \frac{z-3}{z^2+2z+5} dz$ where $C: |z+1-i| = 2$.
   (b) State and Prove Cauchy's integral formula for derivative and use it to evaluate $\int_{|z|=1} \frac{\cos 2}{(2z-1)(z-3)} dz$.
   (c) State and prove Cauchy's theorem for a disk. **(4+5+5)**

2. (a) State and prove Morera's theorem. Also evaluate $\int_C \frac{2z}{z^3-1} dz \quad C: |z| = 3$
   (b) If f(z) is analytic region C of complex plane, then prove that the following statements are equivalent (i) $f^n(a) = 0, \forall\, n = 0,1,2, \ldots$ at a point 'a' in C. (ii) f(z) =0 is a neighbourhood K of a point 'a' in C. (iii) f(z) =0 in C.
   (7+7)

3. (a) Find the radius of convergence of (i) $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$, (ii) $\sum_{n=0}^{\infty} \frac{z^n}{n!\, n^n}$
   (b) Let $f(z) = \sum_{n=1}^{\infty} a_n (z-a)^n$ in $\{|z-a| < R\}$ where R is radius of convergence of the power series. Then prove that the Taylor's expansion of $f(z)$ in the neighborhood of a point 'a' is exactly the given power series.
   (c) Find the Laurent's expansion of $f(z) = \sinh(z + \frac{1}{z})$ for |z| >0. **(4+5+5)**

4. (a) State and prove Taylor's theorem.
   (b) Let $f(z)$ be analytic function having as isolated singularity at $z = a$. If $|f(z)|$ is bounded in a neighbourhood $\{0 < |z-a| < r\}$ then prove that $f(z)$ has a removable singularity at $z = a$.
   (c) State and prove open mapping theorem. **(5+5+4)**

5. Evaluate the following (i) $\int_0^{2\pi} \frac{d\theta}{a+b\,co}$ $(a > b > 0)$ (ii) $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$
   (iii) $\int_{-\infty}^{\infty} \frac{sinx\, dx}{x(x^2-2x+2)}$
   (4+5+5)

6. (a) Outline the argument principle and explain why it is called by that name.
   (b) Let $f(z)$ be a non-constant analytic function in a region D of the complex plane. Then Prove that $|f(z)|$ has no maximum in D.
   (c) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

(4+5+5)

7. (a) State and prove Phragmen Lindel of theorem
   (b) State and prove Weierstrass factorization theorem. **(7+7)**

8. (a) Let f(z) be analytic in the region $|z| < \rho$ and let $z = re^{i\theta}$ be any point of this region. Then prove that $f(re^{i\theta}) = \frac{1}{2\pi}\int_0^{2\pi} \frac{(R^2-r^2)f(Re^{i\theta})d\phi}{R^2-2Rrcos(\theta+\phi)+r^2}$.

   (b) Sate and prove Poisson's-Jensen formula. **(8+6)**

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